for 2 h, 20-mW pulsed (peak power 50 W) for 2 h, and 10-mW CW for 4 h, are indistinguishable. That is, the available data imply that for sensibly continuous exposures, the teratological damage depends upon the total dose received and not upon protocol by which it is applied. Of course, there are biological phenomena which are reciprocal over some ranges but nonreciprocal over others, or are reciprocal for continuously applied stimuli but become nonreciprocal when the dose is applied intermittently over a sufficiently long period of time.[1] Nevertheless, the available data do at least raise the possibility that microwave photon is a cumulative teratogen.

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1 A well-documented example of this is the phototropic response of the coleoptile [4].

The Coupling of a Single-Ridge Waveguide to a Fabry–Perot Resonator

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***Abstract*—The functional forms of the fields in a single-ridge waveguide are presented. Bethe's small-hole diffraction theory is used to determine the coefficient for coupling from various parts of the guide to a Fabry–Perot (FP) resonator. It is shown that frequency-independent coupling over a very broad band is possible, and specific examples are given for the band from 18.0 to 40.0 GHz.**

I. INTRODUCTION

In microwave spectroscopy it is often desired to couple single-mode microwave energy from a waveguide into a resonant cavity over a broad band of frequencies. If this band is wider than the bandwidth of a rectangular waveguide, then it is necessary either to use different sizes of rectangular guides over different parts of the band, or to use another type of guide which has a suitable bandwidth. Using more than one type of guide is inadequate if the input energy is to be continuously swept over the band of frequencies. However, a ridge waveguide is very suitable for this application because of its relatively wide single-mode bandwidth. In this paper, an approximate determination of the fields in a single-ridge waveguide and a scheme for broad-band coupling from a single-ridge waveguide into a Fabry–Perot-type (FP) resonant cavity, with particular emphasis on the band of frequencies from 18.0 to 40.0 GHz, are presented.

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II. DETERMINATION OF BANDWIDTH AND CUTOFF WAVELENGTH

When a rectangular piece of waveguide is loaded with a single ridge down the center (see Fig. 1), the impedance and cutoff frequency are lowered and a wider bandwidth and mode separation are obtained. In order to determine the bandwidth of such a guide, it is necessary to calculate the cutoff wavelengths of the TE_{m0} modes. These wavelengths are usually obtained by assuming parallel-plate TEM modes propagating transversely in the separate rectangular sections of the guide's cross section. The TE_{m0} cutoffs occur at the frequency at which the parallel-plate guide has its mth-order resonance. The discontinuity susceptance B_c, which occurs at the change in height from one region to the other, must be included in the calculation of these resonances. For m-odd modes the resonance must be of the type which gives an infinite impedance at the center of the ridge, while for the m-even modes the ridge-center impedance should be zero.

Fig. 2 shows the equivalent circuit for the ridge guide. From this circuit it is seen that for odd resonances the relation that holds is

$$-Y_{01}\cot\left(\frac{2\pi}{\lambda_c}l\right)+B_c+Y_{02}\tan\left(\frac{2\pi}{\lambda_c}\frac{s}{2}\right)=0 \tag{1}$$

and for even resonances

$$-Y_{01}\cot\left(\frac{2\pi}{\lambda_c}l\right)+B_c-Y_{02}\cot\left(\frac{2\pi}{\lambda_c}\frac{s}{2}\right)=0. \tag{2}$$

In parallel-plate-type waveguides the characteristic impedance of

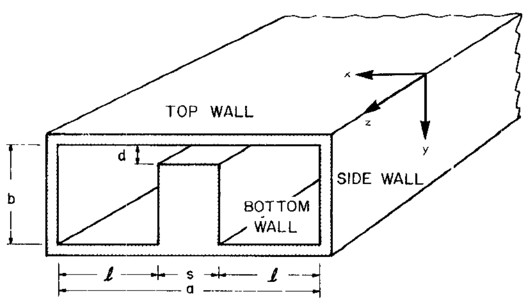

Fig. 1. Configuration of the single-ridge rectangular waveguide showing the coordinate system and the dimensions. a = 0.712 cm; b = 0.3556 cm; s = 0.2667 cm; d = 0.04953 cm; l = 0.2222 cm.

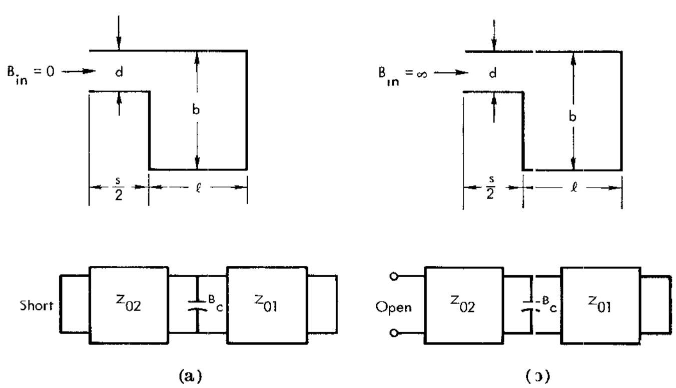

Fig. 2. Equivalent circuits for a single-ridge rectangular waveguide for even and odd modes. (a) For m even. (b) For m odd.

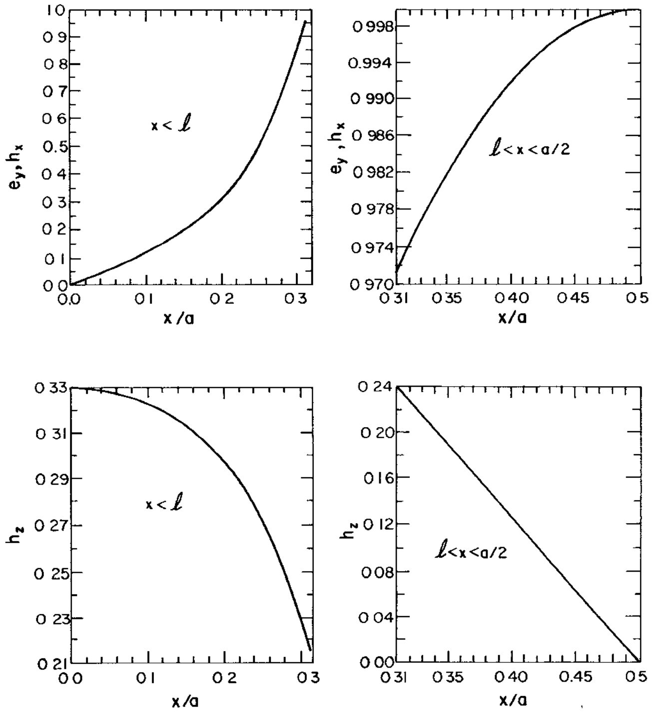

Fig. 3. Peripheral fields along the top wall of waveguide with dimensions shown in Fig. 1.

the guide is proportional to the height of the guide. Thus the preceding equations may be written as

$$\frac{b}{d}\tan\left(\frac{2\pi}{\lambda_c}\frac{s}{2}\right)+\frac{B_c}{Y_{01}}-\cot\left(\frac{2\pi}{\lambda_c}l\right)=0, \qquad (m \text{ odd}) \qquad (3)$$

$$\frac{b}{d}\cot\left(\frac{2\pi}{\lambda_c}\frac{s}{2}\right)-\frac{B_c}{Y_{01}}+\cot\left(\frac{2\pi}{\lambda_c}l\right)=0, \qquad (m \text{ even}). \qquad (4)$$

The term B_c/Y_{01} in the preceding equations is obtainable from a paper by Whinnery and Jamieson [1]. Given the preceding equations, it is then possible to solve them for the cutoff wavelength λ_c. Hopfer [2] has actually solved these equations for single-ridge guide of many dimensions; thus the desired $(\lambda_c)_{m0}$ may be obtained from the graphs in his paper. These graphs were used to obtain the values of $(\lambda_c)_{10} = 0.0349m$ and $(\lambda_c)_{20} = 0.007555m$ for the single-ridge guide of dimensions shown in Fig. 3. From these wavelengths, it can be seen that this particular single-ridge guide has a bandwidth of 8.6–40.0 GHz.

III. FIELD DESCRIPTION

The preceding approach is adequate for finding the cutoff frequencies in the ridge guide, but it does not give a realizable field distribution. Since an exact field distribution in the ridge guide is not feasible, what is needed is a better approximation than that of the TEM parallel-plate guide. The approach used is to again assume a TEM parallel-plate guide mode in the region above the ridge and to match its electric-field component at the edge of the ridge to the electric-field components of a TEM parallel-plate guide mode, plus higher order cutoff TM modes propagating transversely in the large end section. This method gives reasonable results everywhere in the guide except for a small region in the neighborhood of the edge of the ridge. The details of this field derivation have been worked out by Getsinger [3] and may be found in his paper.

Figs. 3–5 show the peripheral fields for a ridge guide with the dimensions used previously in this paper (Fig. 1). The fields were derived using 40 terms of the series expressions for the fields as they

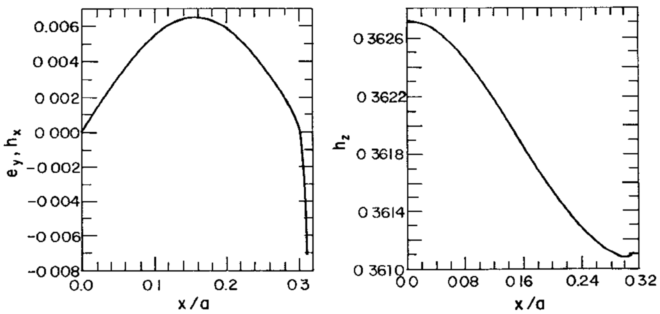

Fig. 4. Peripheral fields along the bottom wall of waveguide with dimensions shown in Fig. 1.

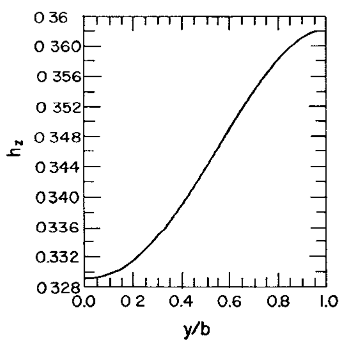

Fig. 5. Peripheral fields along the side wall of waveguide with dimensions shown in Fig. 1.

appeared in Getsinger's paper. The figures show that the fields do not match in the region of the edge of the ridge; in fact, the fields are probably in error in the region from $2.8 < x/a < 3.2$. If one wishes to know the fields more accurately in this region, it is possible to use a static field solution in the region of the edge [3].

IV. DERIVATION OF COUPLING COEFFICIENTS

Once the fields have been determined in the ridge guide, it is a simple matter to calculate the coupling coefficients for launching into a resonant cavity. Here we assume that the energy is coupled from the waveguide to the cavity through a small circular aperture located in any wall of the guide. If this aperture is small in wavelength—specifically, the resonant frequency of the aperture should not be less than three times the operating frequency—it is possible to use Bethe's theory of coupling from small apertures [4]. This approximate theory states that the aperture is equivalent to a combination of radiating electric and magnetic dipoles whose dipole moments are, respectively, proportional to the normal electric field and the tangential magnetic field of the incident wave.

For simplicity in modeling our problem, we assume that only the TE_{10} mode is propagating in the ridge waveguide and that the waveguide is terminated in a matched load so that there are no reflections. Thus propagation is taking place in only one direction. We also assume that we are coupling into a flat parallel-plate FP-type of resonant cavity, and that the fields we are exciting in the cavity are only of the first-order TEM-type, which can be simply written as

$$\bar{H}_r = C\bar{h}_r = C(\hat{u}h_u + \hat{v}h_v) \tag{5a}$$

$$\bar{E}_r = C\bar{e}_r = C(\hat{u}e_u + \hat{v}e_v) \tag{5b}$$

where $\hat{u}$ and $\hat{v}$ are the two unit vectors which are tangential to the plane of the parallel plates (see Fig. 6). The term C in the preceding, which is referred to as the coupling coefficient, is defined by Bethe as

$$C = \frac{j\omega}{2}\left(\mu_0\bar{h}_r\cdot\bar{M} - \bar{e}_r\cdot\bar{P} + \frac{\mu_0}{2}\nabla\bar{h}_r:\bar{Q}\right) \tag{6}$$

where $\bar{M}$ and $\bar{P}$ are the magnetic and electric dipole moments, respectively, and $\bar{Q}$ is a dyadic quadrupole. For our purposes, we can ignore the quadrupole term since it represents a small quantity dependent on the square of the aperture dimension. If a circular aperture is used for coupling then

$$\bar{M} = \alpha_m\cdot\bar{H}_g \tag{7a}$$

$$\bar{P} = \epsilon\alpha_e\hat{n}\hat{n}\cdot\bar{E}_g \tag{7b}$$

where

$$\bar{\alpha}_m = \tfrac{4}{3}(2r)^3(\hat{u}\hat{u} + \hat{v}\hat{v})$$

$$\alpha_e = \frac{-2}{3}(2r)^3$$

where r is the radius of the aperture, and $\bar{H}_g$ and $\bar{E}_g$ are the magnetic and electric fields propagating in the guide.

The expression for the coupling coefficient will be derived for three different locations of the aperture in the guide. Those positions are in the broad wall in the region $l < x < a/2$, in the broad wall in the region $x < l$, and in the narrow wall. These coefficients will be denoted as C_1, C_2, and C_3, respectively.

For coupling from the broad wall of the waveguide, we assume that the transverse fields in the FP cavity are

$$h_v = h_x = -\cos k_0 y \tag{8a}$$

$$h_u = h_z = \cos k_0 y \tag{8b}$$

$$e_v = z_0 \sin k_0 y \tag{8c}$$

$$e_u = -jz_0 \sin k_0 y \tag{8d}$$

where z_0 is the impedance of free space, and the plate separation L in the cavity is $L = n\pi/k_0$, where

$$k_0 = 2\pi/\lambda_0.$$

If Getsinger's expressions for the fields [3] in the region $l < x < a/2$ are substituted into equations (7) then the magnetic and electric dipole moments are

$$\bar{M} = -\hat{x}\tfrac{4}{3}(2r)^3\frac{k_z}{\eta k}\cos\left[k_c\left(\frac{a}{2} - x\right)\right] + \hat{z}\tfrac{4}{3}(2r)^3\frac{jk_c}{\eta k}\sin\left[k_c\left(\frac{a}{2} - x\right)\right] \tag{9a}$$

$$\bar{P} = -\hat{y}\epsilon_0\tfrac{2}{3}(2r)^3\frac{k_z}{\eta k}\cos\left[k_c\left(\frac{a}{2} - x\right)\right] \tag{9b}$$

where

$$k = 2\pi/\lambda$$

$$k_c = 2\pi/\lambda_c$$

$$k_z = \frac{2\pi}{\lambda}[1 - (\lambda/\lambda_c)^2]^{1/2} = \frac{2\pi}{\lambda g}$$

$$\gamma^2 = k_{ny}^2 - k_c^2$$

$$k_{ny}^2 = \frac{n\pi^2}{b}$$

and where λ is the free-space wavelength, λ_c is the cutoff wavelength, and λ_g is the guide wavelength. Thus the coupling coefficient may be written for the aperture in the plane $y = 0$, $l < x < a/2$ as

$$C_1 = j\tfrac{2}{3}(2r)^3\left\{k_z\cos\left[k_c\left(\frac{a}{2} - x\right)\right] - k_c\sin\left[k_c\left(\frac{a}{2} - x\right)\right]\right\}. \tag{10}$$

It should be pointed out that the electric dipole has zero effect since we are not launching any normal electric fields into the cavity.

For the region $x < l$ and $y = 0$ the coefficient C_2 may be similarly

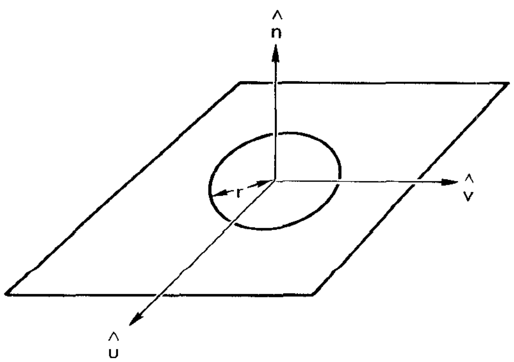

Fig. 6. Circular coupling aperture showing the orientation of the unit vectors.

written as

$$C_2 = j\tfrac{2}{3}(2r)^3 \left\{ k_z \frac{d}{b} \frac{\cos(k_c s/2)}{\sin k_c l} \sin k_c x + k_z \sum_{n=1}^{\infty} \frac{2}{n\pi} \frac{\cos(k_c s/2)}{\sinh \gamma_n l} \sin(n\pi d/b) \sinh \lambda_n x - k_c \cos(k_c s/2) \left[\frac{d}{b} \frac{\cos(k_c x)}{\sin k_c l} - \sum_{n=1}^{\infty} \frac{2}{n\pi} \frac{k_c \sin(n\pi d/b)}{\gamma_n \sinh \gamma_n l} \cosh \gamma_n x \right] \right\} \tag{11}$$

where again the electric dipole has no effect.

When the aperture is in the narrow wall of the guide, the fields in the resonator are of the form

$$h_v = h_y = \cos k_0 x \tag{12a}$$

$$h_u = h_z = j \cos k_0 x \tag{12b}$$

$$e_v = e_y = jz_0 \sin k_0 x \tag{12c}$$

$$e_u = e_z = -z_0 \sin k_0 x. \tag{12d}$$

The magnetic dipole $\bar{M}$ may be expressed as

$$\bar{M} = \frac{4}{3} \frac{(2r)^3}{\eta k} \left\{ \hat{y} k_z \sum_{n=1}^{\infty} \frac{2b}{n\pi} \frac{\gamma_n^2 + k_c^2}{\gamma_n \sinh \gamma_n l} \cdot \cos(k_c s/2) \sin(n\pi d/b) \sin(n\pi y/b) + \hat{z} k_c [\cos(k_c s/2)] \cdot \left[\frac{d}{b \sin k_c l} - \sum_{n=1}^{\infty} \frac{2k_c \sin(n\pi d/b)}{n\pi \gamma_n \sinh \gamma_n l} \cos(n\pi y/b) \right] \right\}. \tag{13}$$

The coupling coefficient C_3 is then

$$C_3 = j\tfrac{2}{3}(2r)^3 [\cos(k_c s/2)] \left[k_z \sum_{n=1}^{\infty} \frac{2b(\gamma_n^2 + k_c^2)}{(n\pi)^2 \gamma_n \sinh \gamma_n l} \sin \frac{n\pi d}{b} \sin \frac{n\pi y}{b} - k_c \left(\frac{d}{b \sin k_c l} - \sum_{n=1}^{\infty} \frac{2k_c \sin(n\pi d/b)}{n\pi \gamma_n \sinh \gamma_n l} \cos \frac{n\pi y}{b} \right) \right]. \tag{14}$$

In (14), the y component of the magnetic field along the narrow wall is much smaller than the z component and can thus be neglected. If the y component is neglected, then the first term in the preceding expression can be removed and the resultant coupling coefficient C_3 is independent of frequency. This is a very important fact if one is interested in obtaining flat coupling from the ridge waveguide into the resonator over the whole band of frequencies. In the expressions for the broad wall coupling coefficients C_1 (10) and C_2 (11) it is seen that if the position x is fixed then the coupling coefficient increases linearly as the frequency increases. The slope of this linear increase increases as the hole is moved toward the center of the wall. If one is only interested in coupling strength and if frequency variation is not important, then the center of the guide is the best place from which to couple.

V. LOSSES DUE TO THE HOLE SIZE

It was pointed out in the beginning of the discussion on the Bethe hole-coupling theory that the aperture diameter was to be much smaller than the wavelength of the highest frequency, but nothing was said about the thickness of the aperture. The thickness of the aperture has an effect on the frequency dependence of the coupling coefficient because the fields will see the aperture as a circular waveguide below cutoff and, thus as an attenuator. The correction term for the coupling coefficient C is thus [5]

$$\exp\left\{ \frac{-2\pi t}{\lambda_{ca}} \left[1 - \left(\frac{\lambda_{ca}}{\lambda} \right)^2 \right]^{1/2} \right\} \tag{15}$$

where λ_{ca} is the cutoff wavelength of the circular aperture and t is the aperture thickness.

VI. CONCLUSIONS

The formulation for the fields in a single-ridge rectangular waveguide has been presented. This formulation was used to derive a theoretical coupling coefficient for coupling from the ridged guide to a FP resonator using a small circular hole. It was pointed out that if coupling was done from the narrow wall of the guide, the coupling coefficient is independent of frequency except for a small variation due to the thickness of the hole. It is also possible to couple from the broad wall, but it was shown that in this case the coupling coefficient was dependent on both position and frequency. The most power may be coupled out when the hole is in the center of the ridge, but at this point the variation of coupling coefficient with frequency is at a maximum. In general, a single-ridge guide with the dimensions listed in Fig. 3 seems theoretically suited to flat coupling over the entire band from 10.0 to 40.0 GHz.

It should be pointed out that the formulation was applied for the case of coupling to the first-order TEM mode in the FP resonator, and thus the results are approximate and need to be confirmed experimentally.

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